The complex nature of route choice models

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Route choice

Objective:

- Understand, describe and predict how travelers select their itinerary in a transportation system,

- within a transportation mode, or across transportation modes.

Applications:

- Real-time operations: Travel information and guidance

- Decision-aid: Traffic simulation

- Policy: congestion pricing

Travel information and guidance

Travel information and guidance

Behavioral issues:

- For prediction: compliance to recommendation
- For guidance: identify preferences to customize the system

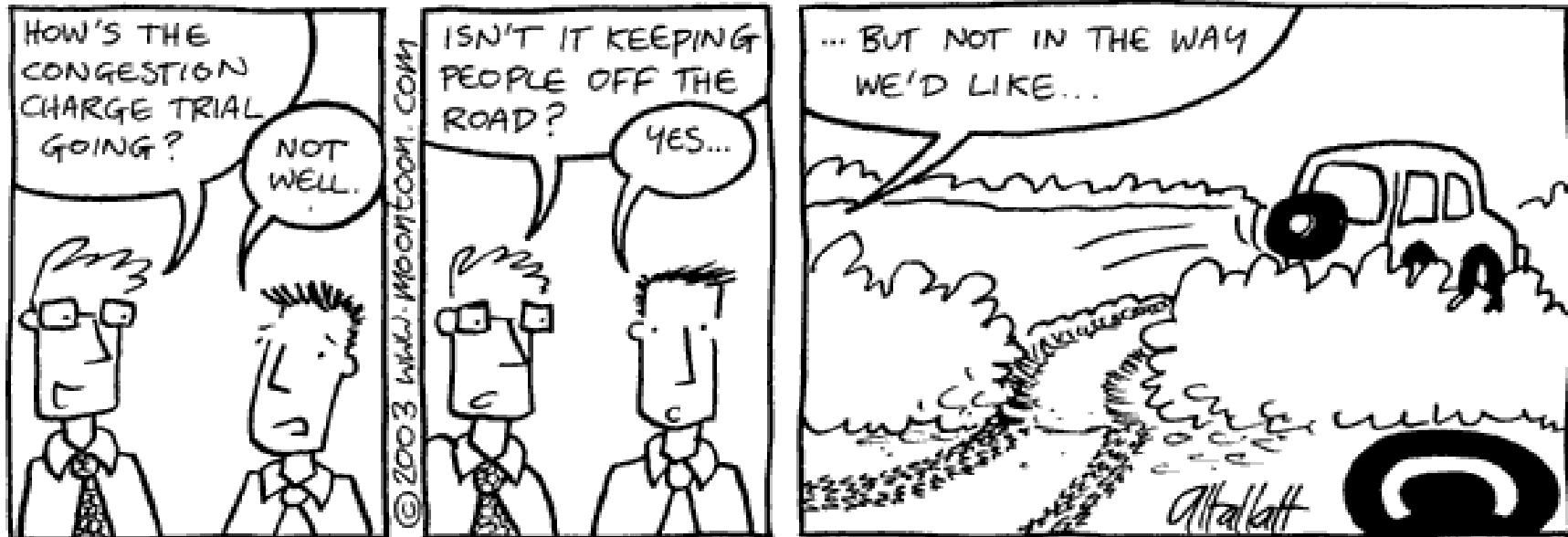# Traffic simulation

Congestion pricing

London

Stockholm

New

York

Congestion pricing

Asking whether congestion pricing is good is like asking 'What is good sex?' It depends on what you do, and if you don't do certain things up front, this will never happen.

Ed Ott (2007)
Executive Director of the New York City Central Labor Council

Complexity of route choice models

Main issues:

- Model
- Data
- Physical overlap of paths
- Large choice set

Emma Frejinger (2008). "Route choice analysis : data, models, algorithms and applications." PhD thesis EPFL, no 4009.

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Route choice model

Given

- a traveler
- a network composed of nodes and directed arcs with attributes
- an origin
- a destination

predict the path chosen by the traveler

Route choice model

Assumptions about

1. the decision-maker: n with characteristics z_n

2. the alternatives
   - Choice set $\mathcal{C}_n$
   - $p \in \mathcal{C}_n$ is composed of a list of links (i,j)

3. the attributes x_{pn}
   - link-additive: length, travel time, etc.

$$(x_{pn})_k = \sum_{(i,j) \in p} \ell_{(i,j)}$$

   - non link-additive: scenic path, usual path, etc.

4. the decision-rules: $\Pr(p|\mathcal{C}_n)$

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Shortest path

Main idea:

- combine the attributes into a generalized cost, possibly traveler-specific

- apply a shortest-path algorithm (Dijkstra)

Disadvantages

- behaviorally irrealistic

- instability with respect to variations in cost

- calibration on real data is very difficult

  - inverse shortest path problem is NP complete

  - Burton, Pulleyblank and Toint (1997) The Inverse Shortest
    Paths Problem With Upper Bounds on Shortest Paths Costs
    Network Optimization , Series: Lecture Notes in Economics
    and Mathematical Systems , Vol. 450, P. M. Pardalos, D.
    W. Hearn and W. W. Hager (Eds.), pp. 156-171, Springer

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Random utility

Main idea:

- The decision-maker n associates a utility U_{pn} with each path p in $\mathcal{C}_n$

- The path with the highest utility is selected

- Utility is an abstract and latent concept which is modeled as a random variable

$$U_{pn} = V_{pn}(x_{pn}, z_n) + \varepsilon_{pn}$$

- Probability model

$$P_n(p|\mathcal{C}_n) = \Pr(U_{pn} \geq U_{qn} \;\; \forall q \in \mathcal{C}_n)$$

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Outline

- Model: **Random utility model**

- Data

- Physical overlap of paths

- Large choice set

Data

Context: Swiss Mobility Pricing Project

- A part of a major study on various mobility pricing scenarios in Switzerland

- A collaboration with ETH Zurich and USI Lugano

- Revealed Preferences (RP) and Stated Preferences (SP) data has been collected

- RP data concern long distance route choice by car
  - Route descriptions are approximative
  - Route choices are latent

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RP Data

- Exact descriptions of chosen routes are difficult and expensive to obtain

- The concept of path and network as we need for modeling is abstract for respondents

- Here, a chosen route is described by a sequence of cities and locations

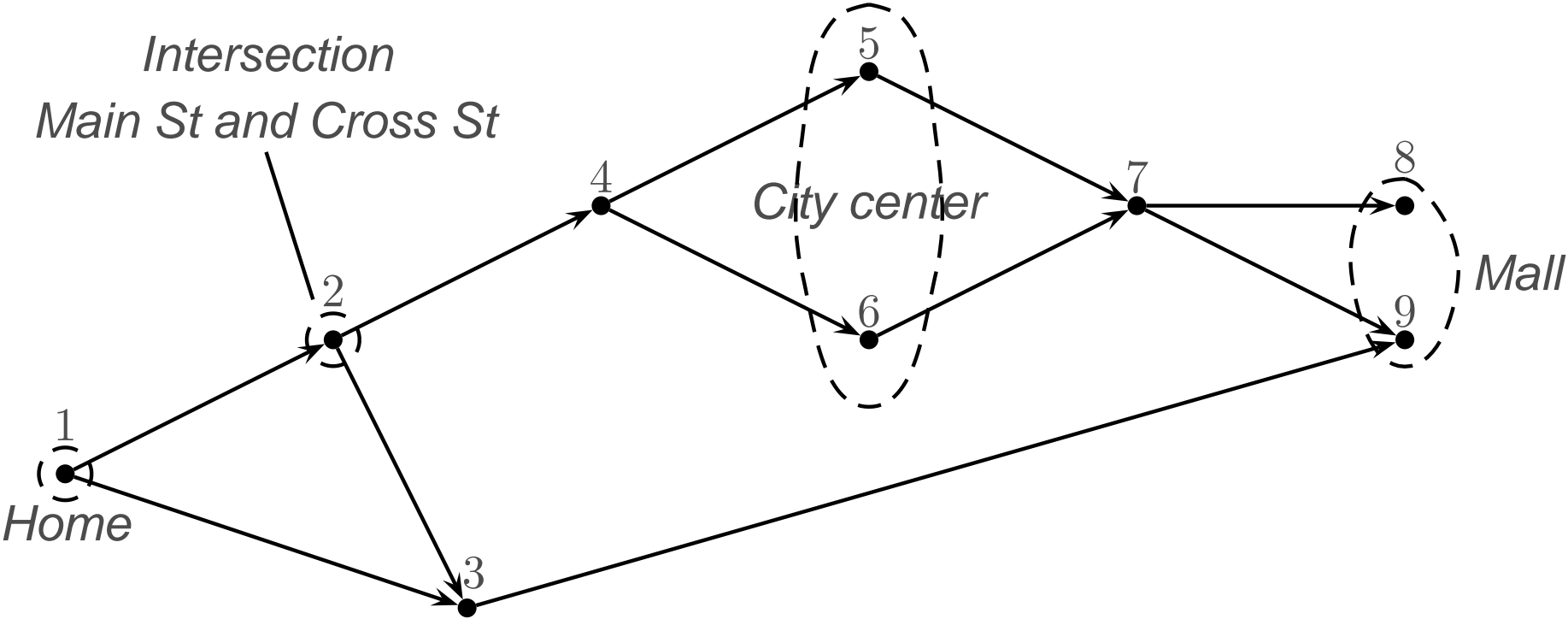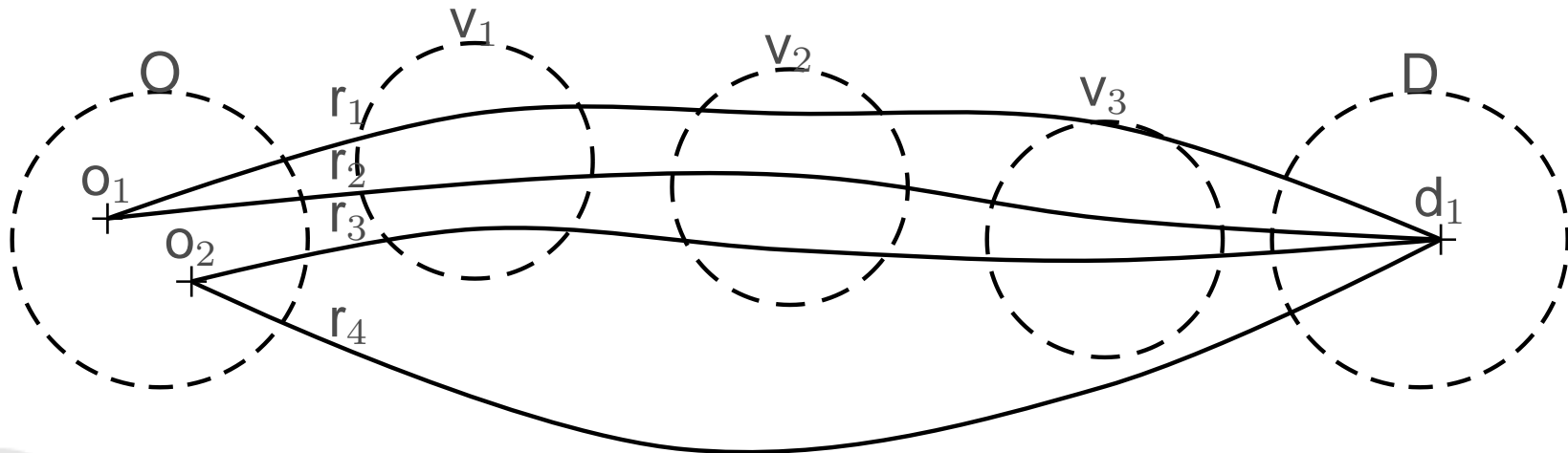- Travelers do not need to refer to the network used by the analyst

RP data

RP data

Intersection
Main St and Cross St

City center

Mall

Home

RP data

Modeling issue:

- *Aggregate observations* (several paths in the network can correspond to the same observation)

- Exact origin-destination pairs are not necessarily known

- Exact route is not known

Similar circumstance: GPS data
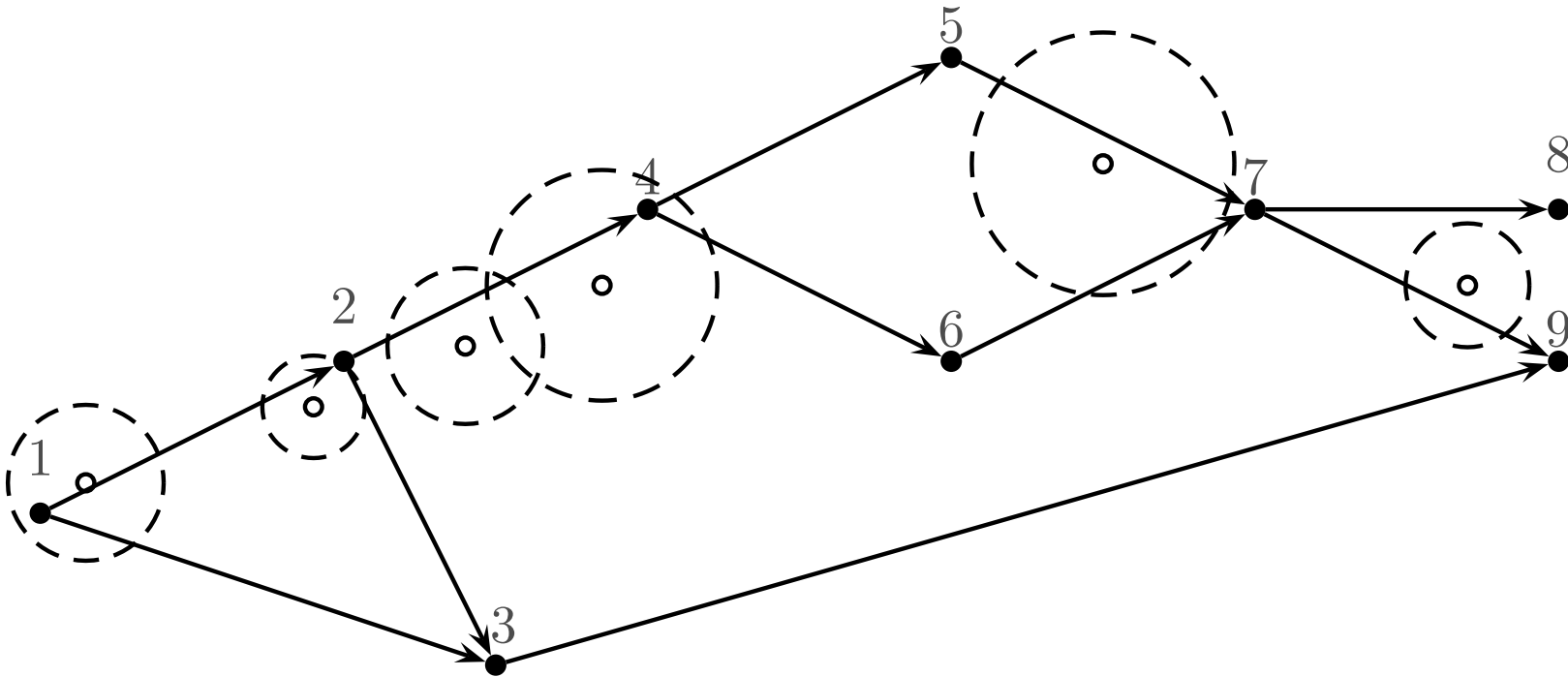
Inappropriate approaches

- "Guess" the path from the aggregate observation
  - Involves subjective judgments and generate noise

- Alternatives in the model are aggregates instead of physical paths
  - Estimated model is of little use in practice

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Proposed approach

Bierlaire, M., and Frejinger, E. (2008). Route choice modeling with network-free data, Transportation Research Part C: Emerging Technologies 16(2):187-198.

Idea:

- The underlying route choice is based on path
- Observations are not based on paths
- Include measurement equations to link the two

Modeling Approach

- Probability of an aggregate observation i:

$$P(i) = \sum_{s \in S} P(s|i) \sum_{r \in C_s} P(i|r) P(r|C_s)$$

- s: origin-destination pair
- S: set of all origin-destination pairs
- r: route
- C_s: set of all routes for origin-destination pair s

Modeling Approach

- Probability of an aggregate observation i:

$$P(i) = \sum_{s \in S} P(s|i) \sum_{r \in C_s} P(i|r)P(r|C_s)$$

  - $P(s|i)$ and $P(i|r)$ can be modeled in several ways
  - $P(r|C_s)$: route choice model that is identifiable if
    1. at least one of the routes in C_s crosses the observed zones, and
    2. at least one route in C_s does not cross the observed zones.
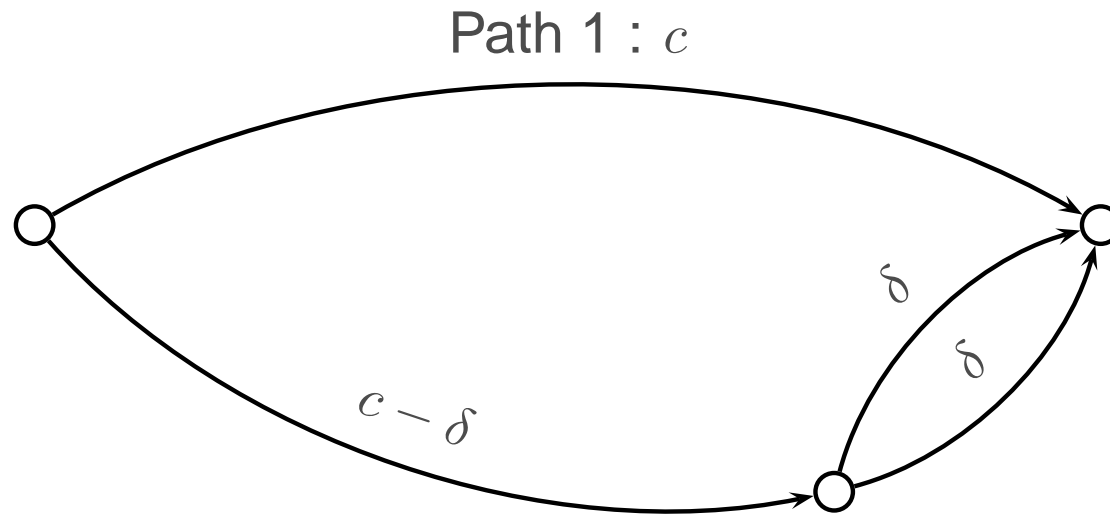- This type of models can be estimated with BIOGEME

Outline

- Model: **Random utility model**

- Data: **Use measurement equations**

- Physical overlap of paths

- Large choice set

Physical overlap

- Assume that the utility of each path is $U_i = -\beta c + \varepsilon_i$

- Assume the logit model (ε_i i.i.d. EV):

$$\Pr(\text{Path 1}) = \frac{e^{-\beta c_1}}{\sum_{q \in \mathcal{C}} e^{-\beta c_q}} = \frac{e^{-\beta c}}{3 e^{-\beta c}} = \frac{1}{3} \text{ for any } c, \delta, \beta$$

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Physical overlap

- Model prediction is counter-intuitive if δ is small

- In this case, the error terms cannot be assumed to be independent

- The model must account for the correlation of the ε due to physical overlap

- Among the existing approaches: path-size and error component models

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Path Size Logit

Ben-Akiva, M., and Bierlaire, M. (2003). Discrete choice models
with applications to departure time and route choice. In Hall, R.
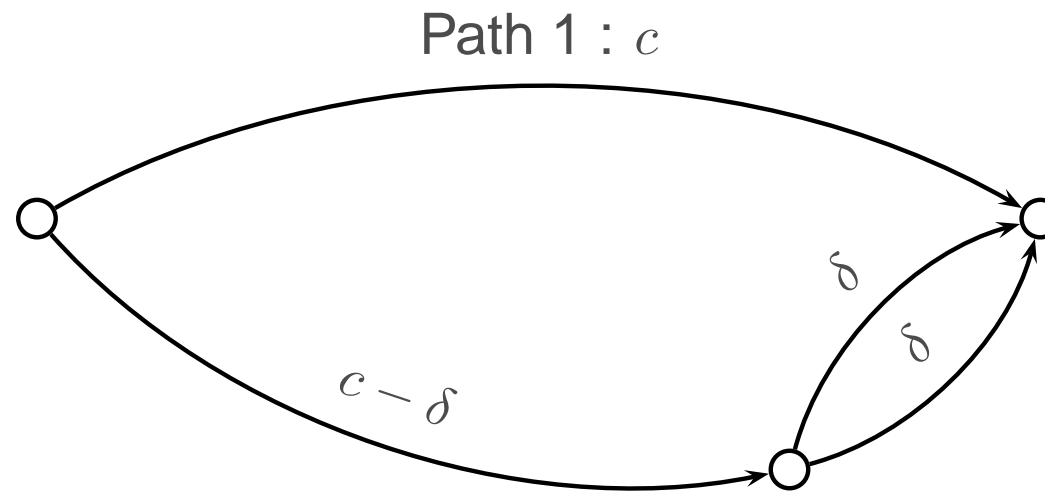(ed) *Handbook of Transportation Science, 2nd edition* pp.7-38.
Kluwer.

- With logit, the utility of overlapping paths is overestimated
- When δ is large, there is some sort of "double counting"
- Idea: include a correction

$$V_p = -\beta c_p + \beta \ln \mathsf{PS}_p$$

where

$$\mathsf{PS}_p = \sum_{(i,j) \in p} \frac{c_{(i,j)}}{c_p} \frac{1}{\sum_{q \in \mathcal{C}} \delta_{i,j}^q} \text{ and } \delta_{i,j}^q = \begin{cases} 1 & \text{if link } (i,j) \text{ belongs to path } q \\ 0 & \text{otherwise} \end{cases}$$
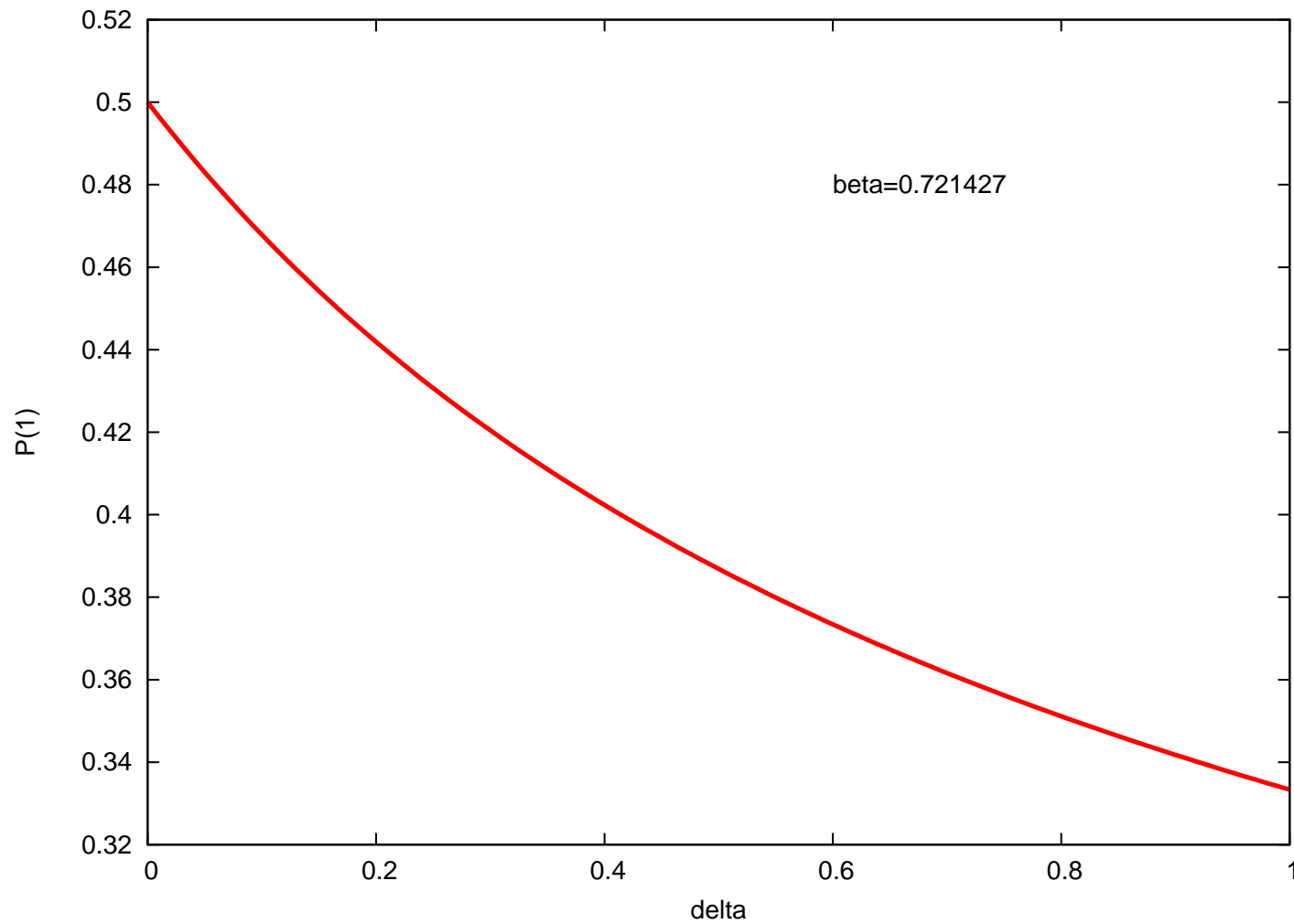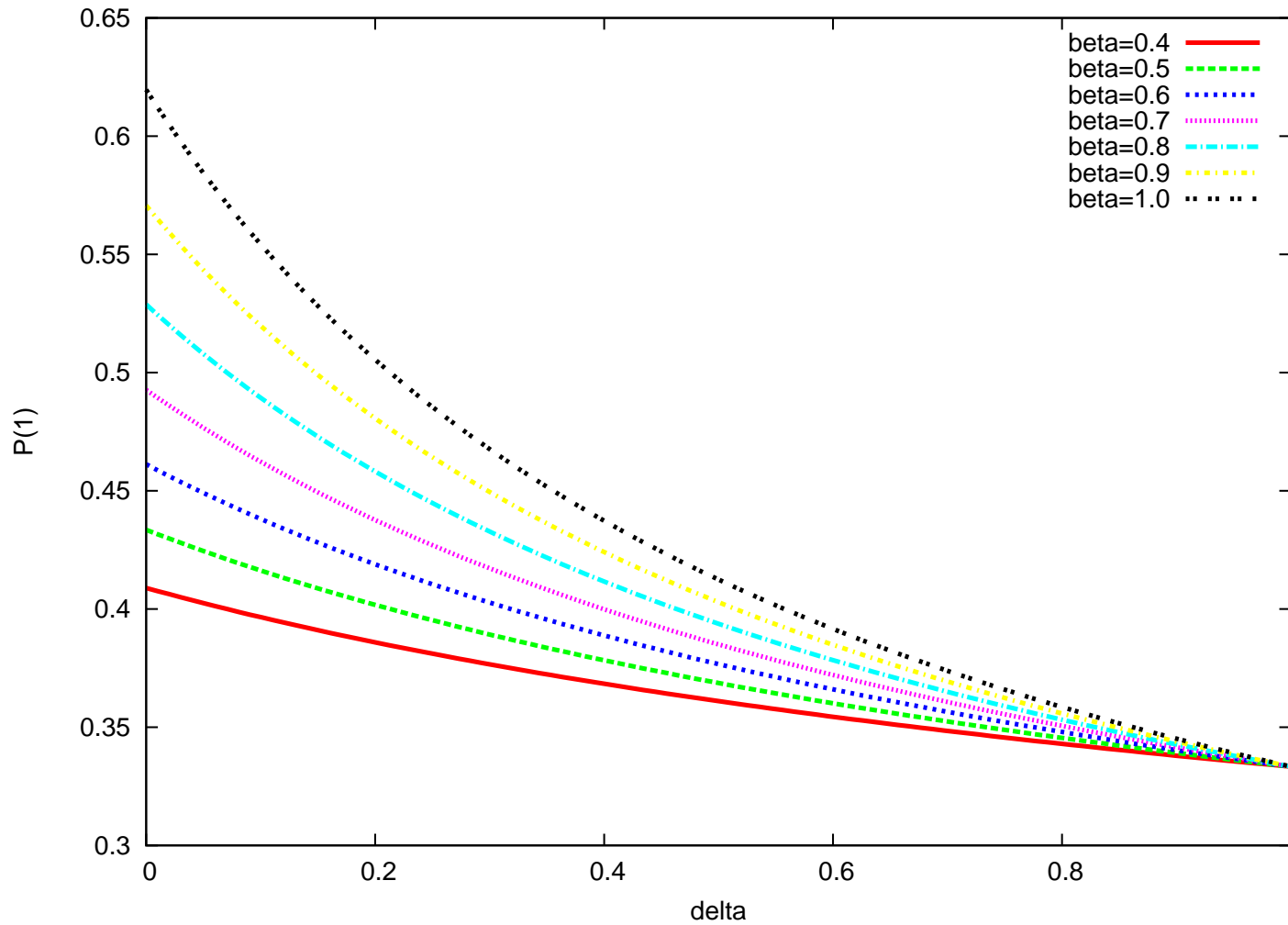
Path Size Logit

Path 1 : c

$$PS_1 \quad = \quad \frac{c}{c}\frac{1}{1} \quad = \quad 1$$

$$PS_2 = PS_3 \quad = \quad \frac{c-\delta}{c}\frac{1}{2} + \frac{\delta}{c}\frac{1}{1} \quad = \quad \frac{1}{2} + \frac{\delta}{2c}$$

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Path Size Logit

beta=0.721427

Path Size Logit

Path Size Logit

Advantages:

- Logit formulation: simple
- Easy to compute
- Exploits the network topology
- Practical

Disadvantages:

- Derived from the theory on nested logit
- Several formulations have been proposed
- Correlated with observed and unobserved attributes
- May give biased estimates

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Logit kernel

Bekhor, Ben-Akiva and Ramming (2002) Adaptation of Logit Kernel to Route Choice Situation. Transportation Research Record. 1805:78-85

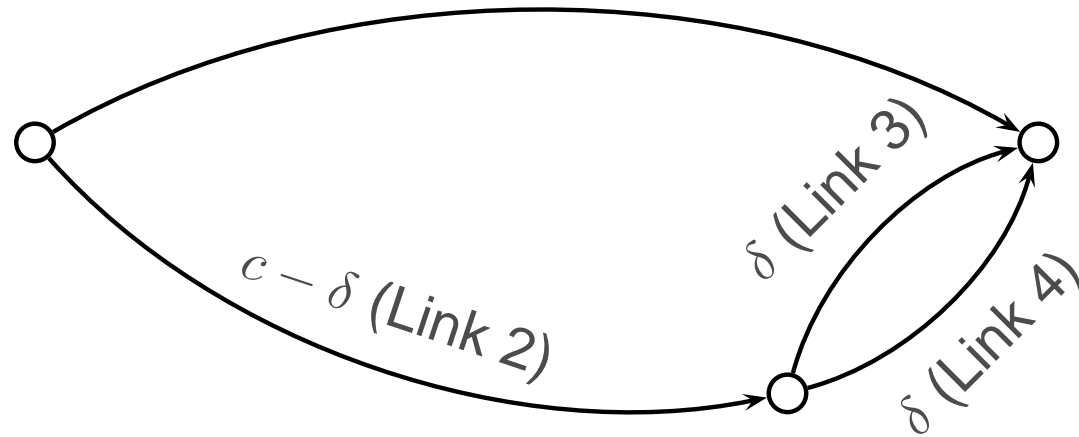Idea: Include an additional error term per link

$$U_p = V_p + \sum_{\ell \in p} f_{\ell p} \sigma_\ell \xi_\ell + \varepsilon_p$$

- $f_{\ell p}$ is a factor loading

- σ_ℓ is a parameter to be estimated (standard deviation)

- ξ_ℓ is a link-specific random term $N(0,1)$

- ε_p is a path-specific random term $EV(0,1)$

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Logit kernel

Path 1 : c (Link 1)

$$
\begin{aligned}
U_{\text{top}} &= -\beta c &&+ \sqrt{c}\sigma_1\xi_1 && && &&+ \varepsilon_{\text{top}} \\
U_{\text{bottomUp}} &= -\beta c && &&+ \sqrt{c-\delta}\sigma_2\xi_2 &&+ \sqrt{\delta}\sigma_3\xi_3 && &&+ \varepsilon_{\text{bottomUp}} \\
U_{\text{bottomDown}} &= -\beta c && &&+ \sqrt{c-\delta}\sigma_2\xi_2 && &&+ \sqrt{\delta}\sigma_4\xi_4 &&+ \varepsilon_{\text{bottomUp}}
\end{aligned}
$$

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Logit kernel

- High number of error components in real networks

- Mixture of logit models has no closed form

- Simulated maximum likelihood estimation is unfeasible with a high number of error components

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Subnetworks

Idea:

- capture the most important correlation structure
- while keeping the model complexity low
- Extend the logit kernel idea
- Apply an error component on *subnetworks* instead of links
- The analyst must define few subnetworks

Subnetworks

How to define the subnetworks?

- A subnetwork component is a set of links corresponding to a part of the network which is behaviorally meaningful and that can be easily labeled

- Subnetwork approach designed to be behaviorally realistic and convenient for the analyst

- Paths sharing a subnetwork component are assumed to be correlated even if they are not physically overlapping

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Subnetworks - Example
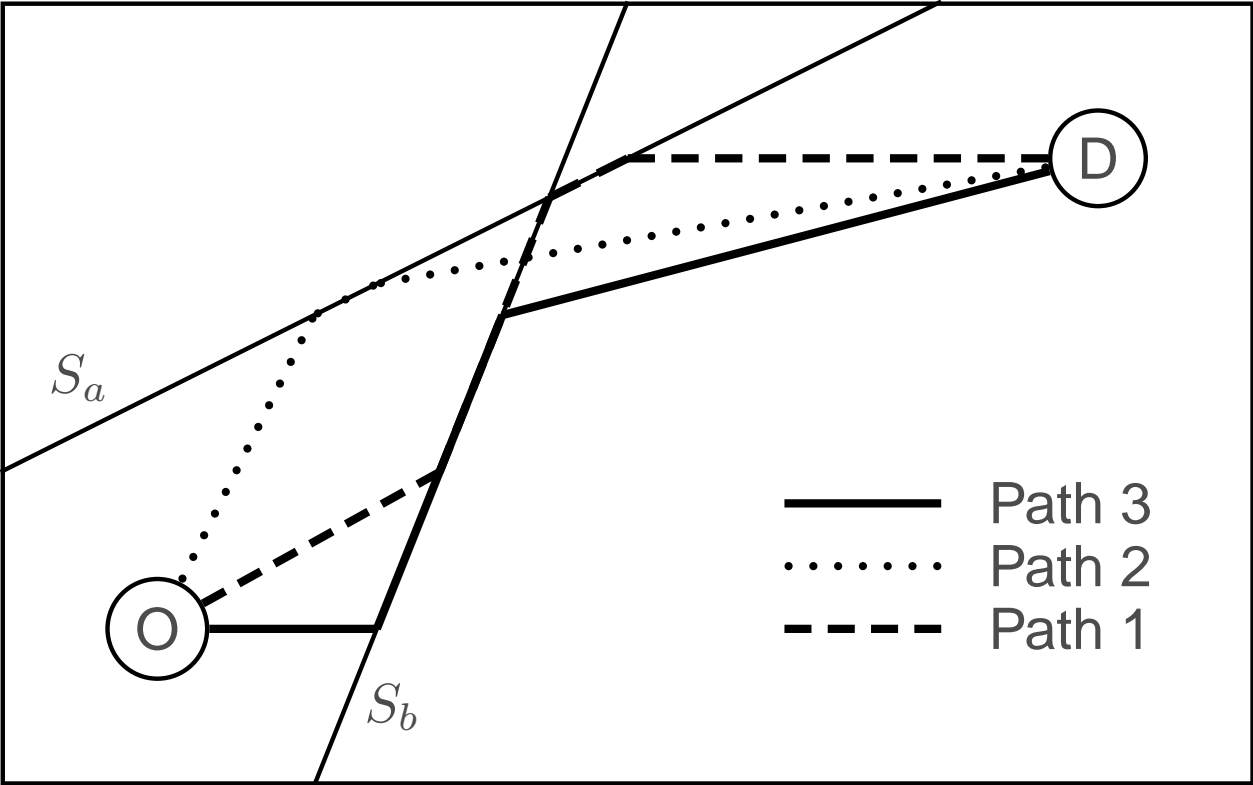
Subnetworks - Methodology

- Factor analytic specification of an error component model (based on model presented in Bekhor et al., 2002)

$$\mathbf{U}_n = \beta^T \mathbf{X}_n + \mathbf{F}_n \mathbf{T} \zeta_n + \varepsilon_n$$

- $\mathbf{F}_{n \ (J \mathrm{x} Q)}$: factor loadings matrix
- $(f_n)_{iq} = \sqrt{l_{niq}}$
- $\mathbf{T}_{(Q \mathrm{x} Q)} = \mathrm{diag}\left(\sigma_1, \sigma_2, \ldots, \sigma_Q\right)$
- $\zeta_{n \ (Q \mathrm{x} 1)}$: vector of i.i.d. N(0,1) variates
- $\varepsilon_{(J \mathrm{x} 1)}$: vector of i.i.d. Extreme Value distributed variates

Subnetworks - Example

- Path 3 (solid line)
- Path 2 (dotted line)
- Path 1 (dashed line)
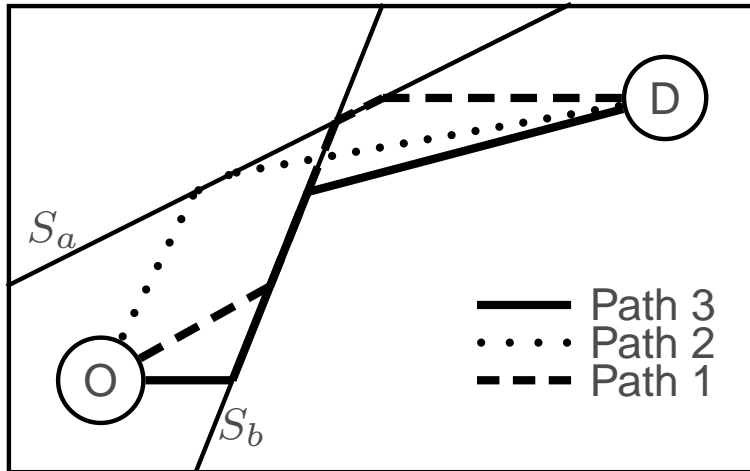
$$U_1 = \beta^T X_1 + \sqrt{l_{1a}}\sigma_a\zeta_a + \sqrt{l_{1b}}\sigma_b\zeta_b + \varepsilon_1$$

$$U_2 = \beta^T X_2 + \sqrt{l_{2a}}\sigma_a\zeta_a + \varepsilon_2$$

$$U_3 = \beta^T X_3 + \sqrt{l_{3b}}\sigma_b\zeta_b + \varepsilon_3$$

$$\mathbf{FTT}^T\mathbf{F}^T =$$

$$\begin{bmatrix} l_{1a}\sigma_a^2 + l_{1b}\sigma_b^2 & \sqrt{l_{1a}}\sqrt{l_{2a}}\sigma_a^2 & \sqrt{l_{1b}}\sqrt{l_{3b}}\sigma_b^2 \\ \sqrt{l_{1a}}\sqrt{l_{2a}}\sigma_a^2 & l_{2a}\sigma_a^2 & 0 \\ \sqrt{l_{3b}}\sqrt{l_{1b}}\sigma_b^2 & 0 & l_{3b}\sigma_b^2 \end{bmatrix}$$

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Empirical Results

- Simplified Swiss network (39411 links and 14841 nodes)

- RP data collection through telephone interviews

- Long distance car travel

- The chosen routes are described with the origin and destination cities as well as 1 to 3 cities or locations that the route pass by

- 940 observations available after data cleaning and verification

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Empirical Results

Empirical Results

- No information available on the exact origin destination pairs

$$P(s|i) = \frac{1}{|S_i|} \ \forall s \in S_i$$

- $P(i|r)$ is modeled with a binary variable

$$\delta_{ri} = \begin{cases} 1 & \text{if } r \text{ corresponds to } i \\ 0 & \text{otherwise} \end{cases}$$

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Empirical Results

- Two origin-destination pairs are randomly chosen for each observation

- 46 routes per choice set are generated with a choice set generation algorithm

- After choice set generation 780 observations are available
    - 160 observations were removed because either all or none of the generated routes crossed the observed zones

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Empirical Results

- Probability of an aggregate observation i

$$P(i) = \sum_{s \in S_i} \frac{1}{|S_i|} \sum_{r \in C_s} \delta_{ri} P(r|C_s)$$

- We estimate Path Size Logit (Ben-Akiva and Bierlaire, 1999) and Subnetwork (Frejinger and Bierlaire, 2007) models

- BIOGEME (biogeme.epfl.ch) used for all model estimations

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Empirical Results - Subnetwork

- Subnetwork: main motorways in Switzerland
- Correlation among routes is explicitly modeled on the subnetwork
- Combined with a Path Size attribute
- Linear-in-parameters utility specifications

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Empirical Results - Subnetwork
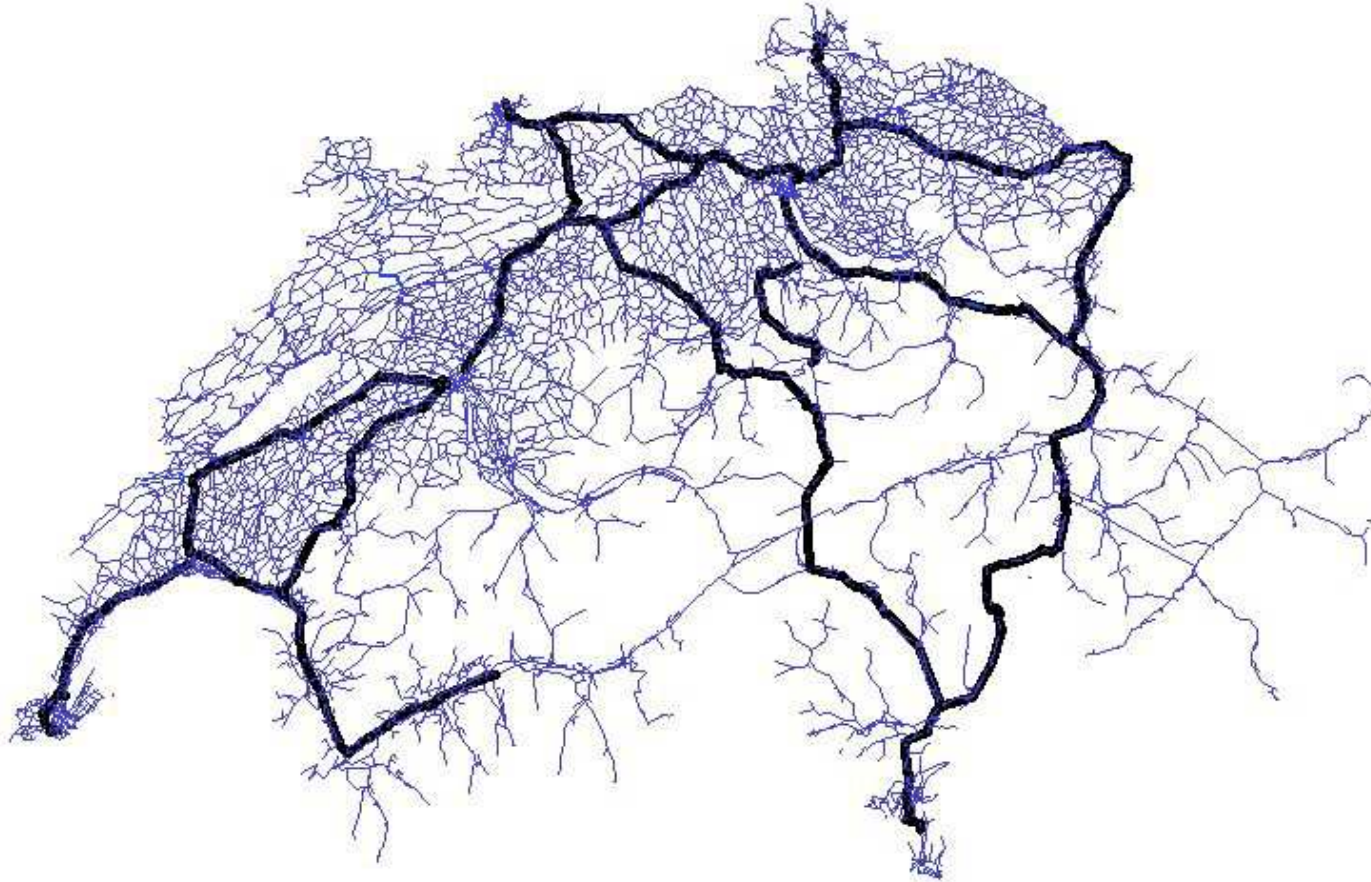
Parameter	PSL		Subnetwork	
In(path size) based on free-flow time	**1.04**	(0.134) 7.81	**1.10**	(0.141) 7.78
Scaled Estimate	*1.04*		*1.04*	
Freeway free-flow time 0-30 min	**-7.12**	(0.877) -8.12	**-7.45**	(0.984) -7.57
Scaled Estimate	*-7.12*		*-7.04*	
Freeway free-flow time 30min - 1 hour	**-1.69**	(0.875) -1.93	**-2.26**	(1.03) -2.19
Scaled Estimate	*-1.69*		*-2.14*	
Freeway free-flow time 1 hour +	**-4.98**	(0.772) -6.45	**-5.64**	(1.00) -5.61
Scaled Estimate	*-4.98*		*-5.33*	
CN free-flow time 0-30 min	**-6.03**	(0.882) -6.84	**-6.25**	(0.975) -6.41
Scaled Estimate	*-6.03*		*-5.91*	
CN free-flow time 30 min +	**-1.87**	(0.331) -5.64	**-2.16**	(0.384) -5.63
Scaled Estimate	*-1.87*		*-2.04*	
Main free-flow travel time 10 min +	**-2.03**	(0.502) -4.05	**-2.46**	(0.624) -3.95
Scaled Estimate	*-2.03*		*-2.33*	
Small free-flow travel time	**-2.16**	(0.685) -3.16	**-2.75**	(0.804) -3.42
Scaled Estimate	*-2.16*		*-2.60*	
Proportion of time on freeways	**-2.2**	(0.812) -2.71	**-2.31**	(0.865) -2.67
Scaled Estimate	*-2.2*		*-2.18*	
Proportion of time on CN	**0 fixed**		**0 fixed**	
Proportion of time on main	**-4.43**	(0.752) -5.88	**-4.40**	(0.800) -5.51
Scaled Estimate	*-4.43*		*-4.16*	
Proportion of time on small	**-6.23**	(0.992) -6.28	**-6.02**	(1.03) -5.83
Scaled Estimate	*-6.23*		*-5.69*	
Covariance parameter			**0.217**	(0.0543) 4.00
Scaled Estimate			*0.205*	

Empirical Results

	PSL	Subnetwork
Covariance parameter		0.217
(Rob. Std. Error) Rob. T-test		(0.0543) 4.00
Number of simulation draws	-	1000
Number of parameters	11	12
Final log-likelihood	-1164.850	-1161.472
Adjusted rho square	0.145	0.147
Sample size: 780, Null log-likelihood: -1375.851		

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Empirical Results

- All parameters have their expected signs and are significantly different from zero

- The values and significance level are stable across the two models

- The subnetwork model is significantly better than the Path Size Logit (PSL) model

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Summary

- Model: **Random utility model**

- Data: **Use measurement equations**

- Physical overlap of paths: **Path Size Logit & Subnetworks**

- Large choice set: `Frejinger, E. (2007). Random Sampling of Alternatives in a Route Choice Context. Proceedings of the European Transport Conference (ETC) October 17-19, 2007`

Illustration: congestion pricing project in Switzerland

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